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Neutrino Oscillations in Supersymmetry without Lepton number conservation and R-parity

Tai-Fu Feng[1,2,3] Xue-Qian Li[1,2,3]

[1]CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, China
[2]Department of Physics, Nankai University, Tianjin 300070, China
[3]Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing 100080, China

Abstract

With the on-shell renormalization scheme, we discuss neutrino masses up to one-loop approximation in the Supersymmetry without lepton number conservation and R-parity. Ii is shown that in this model with experimentally allowed parameters, Δm_{32}^2, Δm_{12}^2 and the mixing angles $|\sin\theta 23|$, $|\sin\theta_{12}|$ which are consistent with the present observation values can be produced. We find that small neutrino mass (≤ 1 eV) sets a loose constraint on the R-parity violation parameters in the soft breaking terms.

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1 Introduction

Nonzero neutrino mass[1] implies that new physics beyond the Minimal Standard Model (MSM) must exist. During the last two decades, the minimal supersymmetric extension of the minimal standard model (MSSM)[2] has been studied carefully. In the supersymmetric model, the renormalizable Lagrangian can include the terms without lepton (L) number and/or baryon (B) number conservation. Usually, we can remove such terms by imposing the R-parity symmetry, with $R = (-1)^{3B+2L+2S}$ (S is the spin of the particle). By this definition, the R-value of the SM particles is $+1$ and that of the supersymmetric partners is -1. Alternatively, we can dismiss the R-parity conservation and retain the B or L violation terms in the Lagrangian. Because the

proton decay data have set a very strict constraint on the B-violation, we presume the B-conservation and only keep an L-number violating interaction in the Lagrangian. In the framework of the supersymmetric model without R-parity, some interesting phenomenological problems such as $\mu \to e\gamma$, neutrino mass and etc. were discussed in [4]. Recently, the supersymmetric model with L-number violation has been employed by many authors to explain the present atmospheric neutrino experiments [3, 6]. The attractive point of the theory is that one generation neutrino can acquire mass through its mixing with gauginos and higgsinos [7] and the other two-generation neutrinos can acquire their masses through loop corrections[8]. This idea urges people to investigate the mechanism which may induce neutrino masses [9]. If in the parameter space one imposes a relation $\epsilon_2 \cos\theta_v + \mu \sin\theta_v \approx 0$ where $\cos\theta_v = v_d/v_L$ and v_d, v_L are the vacuum expectation values of the d-type Higgs and scalar lepton, ϵ_2, μ are the R-parity conserving and violating parameters respectively, he can expect small ν_τ mass due to the generalized SUSY see-saw mechanism. Recently, the authors of [10] have computed the one-loop correction to neutrino masses in the $\overline{MS}$-scheme, they have given a very strict constraint on the parameter space. In this work, we carry out a complete calculation up to one-loop order in the on-shell renormalization scheme and investigate the constraint on the parameter space by the neutrino oscillation data.

It is well known that renormalization is carried out to remove the ultraviolet divergence which appears in the loop calculations. At present, two renormalization schemes are often adopted in the literature. The first is the minimal subtraction (MS) and modified minimal subtraction ($\overline{MS}$) schemes, these schemes are often applied in the QCD-calculations. The reason is that quarks and gluons are confined inside hadrons, no free quark or gluon can be directly observed in experiments, thus their physical masses are not measurable quantities. The second is the on-mass-shell subtraction scheme[11], that is often used in the EW-process calculations. The advantage of the on-shell scheme is that all parameters have clear physical meaning and can be measured directly in experiments. But in general, practical calculations in this scheme are more complicated than in the $\overline{MS}$-scheme. In this paper, we would adopt the on-shell scheme for its advantage. Furthermore, we would perform our calculation in a strict form because any inappropriate approximation may impede us to understand why neutrino mass is so small, unless we have a very strong reason to take any approximation. Our main results can be summarized as follows:

- Using the on-shell renormalization scheme, we find that the loop corrections to the mass matrix elements of neutrino-neutralino are decreasing when masses of the scalar particles turn larger.

- The neutrino oscillation data impose a loose constraint on the parameters of the soft breaking terms.

An extensive analysis of the neutrino oscillations in terms of the supersymmetric model with bilinear R-parity violation is made in [15]. Although a different method is adopted in this work, our results qualitatively coincide with those of [15].

The paper is organized as follows: in Sec.2, we present the basic ingredients of the supersymmetric theory without lepton number conservation and R-parity. Using the on-shell renormalization scheme, the one-loop corrections to the neutrino masses are also included in the section. In Sec.3, we first discuss the one-loop corrections to the neutrino mass with only τ-number violation. Then, we generalize the discussion with three-generation lepton number violation. We close this paper with conclusions and discussions in the last section. Most of the technical details are omitted in the text and then collected in the appendices.

2 The Lagrangian and the on-shell renormalization scheme

In the supersymmetric extension of the standard model without the lepton-number conservation, the down-type Higgs superfield and lepton superfield have the same gauge quantum number, we can combine them into a vector $\hat{L}_J$=($\hat{H}_d$, $\hat{L}_i$) with J=0, 1, 2, 3. Using this notation, the superpotential can be written as

$$W = \mu^J \hat{H}\hat{L}_J + \lambda^{JKl}\hat{L}_J\hat{L}_K E_l^c + \lambda_d^{Jpq}\hat{L}_J\hat{Q}_p\hat{D}_q^c + h_u^{pq}\hat{H}_u\hat{Q}_p\hat{U}_q^c, \tag{1}$$

where $\lambda^{JKl} = -\lambda^{KJl}$. The parameters μ^0, λ^{0kl} and λ_d^{0pq} are the R-parity conserving coupling constants. The other parameters such as μ^i, λ^{kml} and λ_d^{kpq} represent the R-parity violation coupling constants. Here we use the subscripts i, j, k, l, $\cdots$ to represent the generation indices of the leptons and quarks.

In order to break supersymmetry, the following soft terms should be introduced:

$$\begin{aligned} V_{soft} = & \frac{m_{H_u}^2}{2} H_u^\dagger H_u + \frac{1}{2} L^{J\dagger}(m_L^2)_{JK} L^K + B^J H_u L_J \\ & + A_u H_u Q U^c + A_d^J L_J Q D^c + A_e^{JK} L_J L_K E^c + h.c. \end{aligned} \tag{2}$$

Note, here we do not invoke the M-SUGRA scenario and absorb the superpotential parameters into the soft breaking parameters. In terms of W and V_{soft} given in Eq.(1) and Eq.(2), we can compute the loop corrections to the neutrino masses. In this paper, we will perform our analysis in the mass basis, which is independent of the U(4)-rotation among the superfields $\hat{L}_J$. The relevant Feynman rules can be found in [13, 15] and we only cite those contents of the references which are necessary to the calculations of this work.

Now, we compute the one-loop corrections to the neutrino masses in the supersymmetric extension without R-parity. The general form of the self-energy for $\kappa_i^0 - \kappa_j^0$ can be written as

$$\Sigma(k)_{ij} = c_{ij} m_j \omega_- + d_{ij} m_i \omega_+ + e_{ij} \not{k} \omega_- + f_{ij} \not{k} \omega_+. \tag{3}$$

When one external leg of the self-energy is neutrino, $k^2 \ll m_0^2$ with m_0 being the mass of the heaviest internal-particle and we can write c_{ij}, d_{ij}, e_{ij} and f_{ij} as expressions of k^2[12]:

$$\begin{aligned} c_{ij} &= c_{ij}^0 + k^2 c_{ij}^1, \\ d_{ij} &= d_{ij}^0 + k^2 d_{ij}^1, \\ e_{ij} &= e_{ij}^0 + k^2 e_{ij}^1, \\ f_{ij} &= f_{ij}^0 + k^2 f_{ij}^1. \end{aligned} \tag{4}$$

Σ_{ij}'s are renormalized by adding counter-terms and the renormalized Σ_{ij}^{REN} are written as:

$$\Sigma_{ij}^{REN}(k) = \Sigma_{ij}(k) + \left(c_{ij}^* m_j \omega_- + d_{ij}^* m_i \omega_+ + e_{ij}^* \not{k} \omega_- + f_{ij}^* \not{k} \omega_+ \right), \tag{5}$$

where the quantities with * are the counter parts. In the on-shell renormalization scheme they are determined by the mass-shell conditions

$$\begin{aligned} \Sigma_{ij}^{REN}(k) u_i(k)|_{k^2=m_i^2} &= 0, \\ \bar{u}_j(k) \Sigma_{ij}^{REN}(k)|_{k^2=m_j^2} &= 0, \end{aligned} \tag{6}$$

which yield solutions:

$$\begin{aligned} c_{ij}^* &= -c_{ij}^0 + m_i^2 d_{ij}^1 + m_i^2 e_{ij}^1 + m_i m_j f_{ij}^1, \\ d_{ij}^* &= -d_{ij}^0 + m_j^2 c_{ij}^1 + m_j^2 e_{ij}^1 + m_i m_j f_{ij}^1, \\ e_{ij}^* &= -e_{ij}^0 - m_j^2 c_{ij}^1 - m_i^2 d_{ij}^1 - (m_i^2 + m_j^2) e_{ij}^1 - m_i m_j f_{ij}^1, \\ f_{ij}^* &= -f_{ij}^0 - m_i m_j c_{ij}^1 - m_i m_j d_{ij}^1 - m_i m_j e_{ij}^1 - (m_i^2 + m_j^2) f_{ij}^1. \end{aligned} \tag{7}$$

From Eq.5 and Eq.7, the renormalized self-energy is recast into the form

$$\Sigma_{ij}^{REN}(k) = \left(m_i^2 d_{ij}^1 + m_i^2 e_{ij}^1 + m_i m_j f_{ij}^1 + c_{ij}^1 k^2 \right) m_j \omega_-$$

$$+\Big(m_j^2 c_{ij}^1 + m_j^2 e_{ij}^1 + m_i m_j f_{ij}^1 + d_{ij}^1 k^2\Big) m_i \omega_+$$

$$+\Big(- m_j^2 c_{ij}^1 - m_i^2 d_{ij}^1 - (m_i^2 + m_j^2) e_{ij}^1 - m_i m_j f_{ij}^1 + e_{ij}^1 k^2\Big) \not{k}\omega_-$$

$$+\Big(- m_i m_j c_{ij}^1 - m_i m_j d_{ij}^1 - m_i m_j e_{ij}^1 - (m_i^2 + m_j^2) e_{ij}^1 + f_{ij}^1 k^2\Big) \not{k}\omega_+$$

$$= (\not{k} - m_j)\hat{\Sigma}_{ij}(k)(\not{k} - m_i). \tag{8}$$

In the last step, we have written $\Sigma_{ij}^{REN}(k)$ in such a way that their on-shell behavior becomes more obvious as

$$\hat{\Sigma}_{ij}(k) = c_{ij}^1 m_j \omega_+ + d_{ij}^1 m_i \omega_- + e_{ij}^1 (m_i \omega_- + m_j \omega_+ + \not{k}\omega_+)$$

$$+ f_{ij}^1 (m_i \omega_+ + m_j \omega_- + \not{k}\omega_-). \tag{9}$$

For convenience, we introduce some new symbols:

$$\delta Z_{ij}^L = -m_j^2 c_{ij}^1 - m_i^2 d_{ij}^1 - (m_i^2 + m_j^2) e_{ij}^1 - m_i m_j f_{ij}^1 + e_{ij}^1 k^2,$$

$$\delta Z_{ij}^R = -m_i m_j c_{ij}^1 - m_i m_j d_{ij}^1 - m_i m_j e_{ij}^1 - (m_i^2 + m_j^2) f_{ij}^1 + f_{ij}^1 k^2,$$

$$\delta m_{ij}^L = \Big(m_i^2 d_{ij}^1 + m_i^2 e_{ij}^1 + m_i m_j f_{ij}^1 + c_{ij}^1 k^2\Big) m_j,$$

$$\delta m_{ij}^R = \Big(m_j^2 c_{ij}^1 + m_j^2 e_{ij}^1 + m_i m_j f_{ij}^1 + d_{ij}^1 k^2\Big) m_i. \tag{10}$$

Up to one-loop order, the two-point Green's function is

$$\Gamma_{ij}(k) = \Big(\not{k} - m_i^{tree}\Big)\delta_{ij} + \Sigma_{ij}^{REN}(k)$$

$$= \Big(\not{k} - m_i^{tree}\Big)\delta_{ij} + \delta Z_{ij}^L \not{k}\omega_- + \delta Z_{ij}^R \not{k}\omega_+ - \delta m_{ij}^L \omega_- - \delta m_{ij}^R \omega_+$$

$$= (\delta_{ij} + \delta Z_{ij}^L)\Big(\not{k} - m_i^{tree} - \delta m_{ij}^L + \delta Z_{ij}^L m_i^{tree}\Big)\omega_-$$

$$+(\delta_{ij} + \delta Z_{ij}^R)\Big(\not{k} - m_i^{tree} - \delta m_{ij}^R + \delta Z_{ij}^R m_j^{tree}\Big)\omega_+, \tag{11}$$

where $\delta_{ij} + \delta Z_{ij}^L$ is the renormalization multiplier for the left-handed wave function and $\delta_{ij} + \delta Z_{ij}^R$ is the renormalization multiplier for the right-handed wave function. m_i^{tree} is the mass of the i-th generation of fermions at tree level. From Eq.11 and the mass-shell conditions, we derive the loop corrections to the mass matrix elements as:

$$\delta m_{ij}^{loop} = \Big\{ \Big[\delta m_{ij}^L + \delta m_{ij}^R\Big]_{k^2=0} - \Big[m_i \delta Z_{ij}^L|_{k^2=m_i^2} + m_j \delta Z_{ij}^R|_{k^2=m_j^2}\Big]\Big\}$$

$$= 3m_i^{tree}(m_j^{tree})^2 c_{ij}^1 + (m_i^{tree} m_j^{tree} + (m_i^{tree})^2 + (m_j^{tree})^2) m_i^{tree} d_{ij}^1$$

$$+((m_i^{tree})^2 m_j^{tree} + 3m_i^{tree}(m_j^{tree})^2) e_{ij}^1 + (3(m_i^{tree})^2 m_j^{tree} + m_i^{tree}(m_j^{tree})^2) f_{ij}^1, \tag{12}$$

in which $\delta Z_{ij}^{L,R}$, $\delta m_{ij}^{L,R}$ are defined in Eq.10. Note that the above formulae are correct for any type of fermions. Eq.12 is the key formulation to compute the one-loop corrections for the mass matrix of neutrino-neutralino.

In order to compute the form factors $c_{ij}^{0,1}$, $d_{ij}^{0,1}$, $e_{ij}^{0,1}$ and $f_{ij}^{0,1}$, the one-loop self energy diagrams should be precisely calculated. The exchanged bosons in the diagrams can be either vector or scalar and they correspond to different integrals. The integral for exchanging vector-boson is

$$\begin{aligned}
Amp_v(k) &= (\mu_w)^{2\epsilon} \int \frac{d^D Q}{(2\pi)^D} (iA_{\sigma_1}^{(V)} \gamma_\mu \omega_{\sigma_1}) \frac{i(\not{Q} + \not{k} + m_f)}{(Q+k)^2 - m_f^2} (iB_{\sigma_2}^{(V)} \gamma^\mu \omega_{\sigma_2}) \frac{-i}{Q^2 - m_V^2} \\
&= -\int_0^1 dx \int \frac{d^D Q}{(2\pi)^D} \frac{1}{(Q^2 + x(1-x)k^2 - x m_f^2 - (1-x) m_V^2)^2} \Big\{ (2-D) A_\sigma^{(V)} B_\sigma^{(V)} (1-x) \not{k} \omega_\sigma \\
&\quad + D m_f A_{\bar{\sigma}}^{(V)} B_\sigma^{(V)} \omega_\sigma \Big\} \\
&= -i \int_0^1 dx \int \frac{d^D Q}{(2\pi)^D} \frac{1}{(Q^2 + x m_f^2 + (1-x) m_V^2)^2} \Big\{ 1 + \frac{2x(1-x)k^2}{Q^2 + x m_f^2 + (1-x) m_V^2} \Big\} \\
&\quad \Big\{ (2-D) A_\sigma^{(V)} B_\sigma^{(V)} (1-x) \not{k} \omega_\sigma + D m_f A_{\bar{\sigma}}^{(V)} B_\sigma^{(V)} \omega_\sigma \Big\}
\end{aligned} \tag{13}$$

where $D = 4 - 2\epsilon$ and μ_w represents the renormalization scale. $A_\sigma^{(V)}$, $B_\sigma^{(V)}$ with $\sigma = \pm$ are the interaction vertices. m_V represents the mass of the vector boson that appears in the loop and m_f is for the fermion in the loop. From Eq.3, Eq.4 and Eq.13, we get

$$\begin{aligned}
c_{ij}^0(m_V, m_f) &= -iD \frac{m_f}{m_j} A_+^{(V)} B_-^{(V)} F_{2a}(m_f, m_V), \\
d_{ij}^0(m_V, m_f) &= -iD \frac{m_f}{m_i} A_-^{(V)} B_+^{(V)} F_{2a}(m_f, m_V), \\
e_{ij}^0(m_V, m_f) &= -i(2-D) A_-^{(V)} B_-^{(V)} F_{2b}(m_f, m_V), \\
f_{ij}^0(m_V, m_f) &= -i(2-D) A_+^{(V)} B_+^{(V)} F_{2b}(m_f, m_V), \\
c_{ij}^1(m_V, m_f) &= -i4 \frac{m_f}{m_j} A_+^{(V)} B_-^{(V)} F_{3a}(m_f, m_V), \\
d_{ij}^1(m_V, m_f) &= -i4 \frac{m_f}{m_i} A_-^{(V)} B_+^{(V)} F_{3a}(m_f, m_V), \\
e_{ij}^1(m_V, m_f) &= i2 A_-^{(V)} B_-^{(V)} F_{3b}(m_f, m_V), \\
f_{ij}^1(m_V, m_f) &= i2 A_+^{(V)} B_+^{(V)} F_{3b}(m_f, m_V).
\end{aligned} \tag{14}$$

F_{2a}, F_{2b}, F_{3a} and F_{3b} are integrals over the internal momentum of the loop and their explicit forms are given in appendix A.

For exchanging scalar-boson, the amplitude is derived in a similar way and it is

$$
\begin{aligned}
Amp_s(k) &= (\mu_w)^{2\epsilon} \int \frac{d^D Q}{(2\pi)^D} (iA^{(S)}_{\sigma_1}\omega_{\sigma_1}) \frac{i(\not{Q} + \not{k} + m_f)}{(Q+k)^2 - m_f^2} (iB^{(S)}_{\sigma_2}\omega_{\sigma_2}) \frac{i}{Q^2 - m_S^2} \\
&= i \int_0^1 dx \int \frac{d^D Q}{(2\pi)^D} \frac{1}{(Q^2 + x m_f^2 + (1-x) m_S^2)^2} \Big\{ 1 + \frac{2x(1-x)k^2}{Q^2 + x m_f^2 + (1-x) m_S^2} \Big\} \\
&\quad \Big\{ A^{(S)}_{\bar{\sigma}} B^{(S)}_{\sigma} (1-x) \not{k} \omega_\sigma + m_f A^{(S)}_{\sigma} B^{(S)}_{\sigma} \omega_\sigma \Big\},
\end{aligned}
\tag{15}
$$

where $A^{(S)}_\sigma$, $B^{(S)}_\sigma$ with $\sigma = \pm$ are the interaction vertices. m_S represents the mass of the scalar boson that appears in the loop and m_f is for the fermion in the loop. From Eq.3, Eq.4 and Eq.15, we obtain

$$
\begin{aligned}
c^0_{ij}(m_S, m_f) &= i \frac{m_f}{m_j} A^{(S)}_- B^{(S)}_- F_{2a}(m_f, m_S), \\
d^0_{ij}(m_S, m_f) &= i \frac{m_f}{m_i} A^{(S)}_+ B^{(S)}_+ F_{2a}(m_f, m_S), \\
e^0_{ij}(m_S, m_f) &= i A^{(S)}_+ B^{(S)}_- F_{2b}(m_f, m_S), \\
f^0_{ij}(m_V, m_f) &= i A^{(S)}_- B^{(S)}_+ F_{2b}(m_f, m_S), \\
c^1_{ij}(m_S, m_f) &= i \frac{m_f}{m_j} A^{(S)}_- B^{(S)}_- F_{3a}(m_f, m_S), \\
d^1_{ij}(m_S, m_f) &= i \frac{m_f}{m_i} A^{(S)}_+ B^{(S)}_+ F_{3a}(m_f, m_S), \\
e^1_{ij}(m_S, m_f) &= i A^{(S)}_+ B^{(S)}_- F_{3b}(m_f, m_S), \\
f^1_{ij}(m_V, m_f) &= i A^{(S)}_- B^{(S)}_+ F_{3b}(m_f, m_S).
\end{aligned}
\tag{16}
$$

In the supersymmetric extension of the SM, the mixing of $\kappa^0_i \sim \kappa^0_j$ originates from the following loop-diagrams.

- The internal particles are Z ~gauge boson and neutralinos (or neutrinos through mixing) κ^0_α ($\alpha = 1, 2, \cdots, 7$).
- The internal particles are W ~gauge boson and charginos (or charged leptons) κ^0_α ($\alpha = 1, 2, \cdots, 5$).
- The internal particles are CP-even Higgs bosons H^0_β ($\beta = 1, 2, \cdots, 5$) and neutralinos (or neutrinos through mixing) κ^0_α ($\alpha = 1, 2, \cdots, 7$).

- The internal particles are CP-odd Higgs bosons A^0_β ($\beta = 1, 2, \cdots, 5$) and neutralinos (or neutrinos through mixing) κ^0_α ($\alpha = 1, 2, \cdots, 7$).

- The internal particles are charged Higgs bosons H^+_β ($\beta = 1, 2, \cdots, 8$) and charginos (or charged leptons) κ^-_α ($\alpha = 1, 2, \cdots, 5$).

- The internal particles are up-type scalar quarks $\tilde{U}^k_\alpha$ ($\alpha = 1, 2$; k=1, 2, 3.) and quark u^k.

- The internal particles are down-type scalar quarks $\tilde{D}^k_\alpha$ ($\alpha = 1, 2$; k=1, 2, 3.) and quark d^k.

The expressions of the contributions from those loop diagrams to the self-energy are presented in Appendix B.

Summing over Eq.29, Eq.31, Eq.34, Eq.37, Eq.40, Eq.42 and Eq.44 in Appendix B, we obtain the one-loop corrections to the neutrino-neutralino mass matrix

$$\delta m_{ij}^{1-loop} = \delta m_{ij}^{(Z,\kappa^0)} + \delta m_{ij}^{(W,\kappa^-)} + \delta m_{ij}^{(H^0,\kappa^0)} + \delta m_{ij}^{(A^0,\kappa^0)} + \delta m_{ij}^{(H^+,\kappa^-)} + \delta m_{ij}^{(\tilde{U},u)} + \delta m_{ij}^{(\tilde{D},d)}. \qquad (17)$$

From the above analysis, we find that the one-loop corrections to the neutrino-neutralino mass matrix are decreasing when m_S (S represents the scalar SUSY or Higgs particles) turns heavier. As a pioneer work, the authors of [4] discussed the neutrino problems in the supersymmetric model without R-parity. Even though our method is different from theirs, our results are qualitatively consistent with theirs, namely, as the mass of the SUSY particles increases, the mass of neutrinos decreases.

3 The masses of neutrinos

In this section, we discuss the diagonalization of the mass matrix of neutrino-neutralino with one-loop correction. For illustrating the physics picture, we first consider a simplified model where only τ-number is violated, and then generalize the result to the case of three-generation lepton number violation.

3.1 A simplified model: only the τ-number is violated

In the mass basis, the one-loop correction to the neutrino-neutralino can be written as

$$\mathcal{M}_N^{1-loop} = \begin{pmatrix} m^{tree}_{\kappa^0_1} & 0 & 0 & 0 & \delta m_{15} \\ 0 & m^{tree}_{\kappa^0_2} & 0 & 0 & \delta m_{25} \\ 0 & 0 & m^{tree}_{\kappa^0_3} & 0 & \delta m_{35} \\ 0 & 0 & 0 & m^{tree}_{\kappa^0_4} & \delta m_{45} \\ \delta m_{15} & \delta m_{25} & \delta m_{35} & \delta m_{45} & m^{tree}_{\nu_\tau} + \delta m_{55} \end{pmatrix}. \qquad (18)$$

In fact, for neutralinos are much heavier than neutrinos and the loop-corrections to them are very small, so that here, we have ignored the loop corrections to the masses of neutralinos. From Eq.18, we can get

$$m_{\nu_\tau}^{1-loop} = m_{\nu_\tau}^{tree} + \delta m_{55} - \frac{\delta m_{15}^2}{m_{\kappa_1^0}} - \frac{\delta m_{25}^2}{m_{\kappa_2^0}} - \frac{\delta m_{35}^2}{m_{\kappa_3^0}} - \frac{\delta m_{45}^2}{m_{\kappa_4^0}}. \tag{19}$$

All the off-diagonal mass terms and extra diagonal masses δm_{ij} are given in Eq.(17). Different from the results in the $\overline{MS}$-scheme and the approach with the mass-insertion method, the on-shell scheme may give rise to larger corrections to the off-diagonal matrix elements $\delta m_{ij}(i \neq j)$ than to the diagonal elements for small neutrino masses. We will give more discussions on the difference in the last section. The corrections from W,Z-gauge bosons and Goldstone particles are suppressed by the small mixing between the neutrino-neutralino. Now, we discuss the scalar particle contributions. For convenience, we restrain our discussion in the basis where $\upsilon_L = 0$ (the vacuum expectation value of the scalar lepton). When the CP-even Higgs mass $m_{H_\beta^0} \gg m_{\kappa_\alpha^0}$, the off-diagonal correction from H_β^0 is

$$\begin{aligned}
\delta m_{i5} \sim & \frac{1}{(4\pi)^2} \frac{m_{\kappa_i^0}^2}{2m_{H_\beta^0}^2} (c_w Z_N^{j2} - s_w Z_N^{j1}) \Big(\sum_{\delta=1}^{5} Z_E^{\beta\delta} Z_N^{i(2+\delta)} - 2 Z_E^{\beta 2} Z_N^{i3} \Big) \Big[c_w Z_E^{\beta 2} \frac{1}{2} g \upsilon_1 \\
& + c_w Z_E^{\beta 1} \frac{1}{2} g \upsilon_2 + s_w Z_E^{\beta 2} \frac{1}{2} g' \upsilon_1 + s_w Z_E^{\beta 1} \frac{1}{2} g' \upsilon_2 \Big] \frac{e^2}{4 s_w^2 c_w^2} \\
\sim & \frac{\alpha_e}{32\pi} \frac{m_{\kappa_i^0}^2 m_w}{m_{H_\beta^0}^2}.
\end{aligned} \tag{20}$$

As $\frac{\delta m_{i5}^2}{m_i} \sim 10^{-9}$(GeV) is required, we have $m_{H_\beta^0} \geq 3 m_i$. The corrections from the physical CP-odd Higgs bosons, Charged Higgs bosons can be discussed in a similar way, and they are suppressed by the large masses of the scalar bosons. For small neutrino masses, one could have another solution where all the R-parity violation parameters (in soft breaking terms and the superpotential) are all small, thus their contributions can be neglected, this scenario has been discussed by Davidson *et.al* [10].

3.2 The model with three-generation lepton number violation

In this case, the mass matrix of neutrino-neutralino can be written as

$$\mathcal{M}_N^{1-loop} = \begin{pmatrix}
m_{\kappa_1^0}^{tree} & 0 & 0 & 0 & \delta m_{15} & \delta m_{16} & \delta m_{17} \\
0 & m_{\kappa_2^0}^{tree} & 0 & 0 & \delta m_{25} & \delta m_{26} & \delta m_{27} \\
0 & 0 & m_{\kappa_3^0}^{tree} & 0 & \delta m_{35} & \delta m_{36} & \delta m_{37} \\
0 & 0 & 0 & m_{\kappa_4^0}^{tree} & \delta m_{45} & \delta m_{46} & \delta m_{47} \\
\delta m_{15} & \delta m_{25} & \delta m_{35} & \delta m_{45} & m_{\nu_\tau}^{tree} + \delta m_{55} & \delta m_{56} & \delta m_{57} \\
\delta m_{16} & \delta m_{26} & \delta m_{36} & \delta m_{46} & \delta m_{56} & 0 & 0 \\
\delta m_{17} & \delta m_{27} & \delta m_{37} & \delta m_{47} & \delta m_{57} & 0 & 0
\end{pmatrix}. \tag{21}$$

The eigenequation of the matrix (21) is

$$x^3-\Big(m^{tree}_{\nu_\tau}+\delta m_{55}-\sum_{i=1}^{4}\frac{\delta m_{i5}^2+\delta m_{i6}^2+\delta m_{i7}^2}{m^{tree}_{\kappa_i^0}}\Big)x^2$$
$$-\Big(\sum_{i=1}^{4}\frac{(\delta m_{i6}^2+\delta m_{i7}^2)(m^{tree}_{\nu_\tau}+\delta m_{55})-2\delta m_{i5}\delta m_{i6}\delta m_{56}-2\delta m_{i5}\delta m_{i7}\delta m_{57}}{m^{tree}_{\kappa_i^0}}$$
$$+(\delta m_{56}^2+\delta m_{57}^2)\Big)x-\sum_{i=1}^{4}\frac{(\delta m_{i7}\delta m_{56}-\delta m_{i6}\delta m_{57})^2}{m^{tree}_{\kappa_i^0}}=0. \tag{22}$$

It is noted that even though M_N^{1-loop} is a 7th-order matrix, because of the form of the up-left corner submatrix, this equation can be reduced into a 3rd-order one.

In a general theory, one can notice from the analysis given above that the lepton-number violation terms (bilinear or trilinear) and the corresponding soft-violating terms affect the mass matrix of neutrinos through mixings of neutrino-neutralino, charged lepton-chargino and Higgs-slepton. However, in this general scenario, there are too many parameters to reduce the prediction power of the model. Thus in our later analysis, we ignore the trilinear terms $\lambda^{ikl}\hat{L}_i\hat{L}_k\hat{E}_l^c$ $(i,k,l=1,2,3)$, $\lambda_d^{jpq}\hat{L}_j\hat{Q}_p\hat{D}_q^c$ $(j,p,q=1,2,3)$ in the superpotential and the corresponding soft-violating terms.

In terms of the mass insertion method, the authors of [16] discussed the modification to the neutrino mass caused by influence of the R-parity violating trilinear terms of the superpotential in every details. For example, if only the third generation of the down-type scalar quarks is relatively light, the modification of the $q\tilde{q}-$loops induced by the R-parity violating trilinear terms and the corresponding soft-violating terms of the superpotential to the neutrino mass matrix is

$$\delta m^{q\tilde{q}}_{(8-i)(8-j)}\sim\frac{3\lambda_d^{i33}\lambda_d^{j33}}{8\pi^2}\frac{m_b^2(A_d^3+\mu^0\tan\beta)}{m_{\tilde{b}}^2}. \tag{23}$$

Similarly, if the third generation of charged sleptons is the lightest, the contribution of $l\tilde{l}-$loops induced by $\lambda^{ijk}\hat{L}_i\hat{L}_j\hat{E}_k^c$ and the corresponding soft-violating terms is

$$\delta m^{l\tilde{l}}_{(8-i)(8-j)}\sim\frac{3\lambda^{i33}\lambda^{j33}}{8\pi^2}\frac{m_\tau^2(A_l^3+\mu^0\tan\beta)}{m_{\tilde{\tau}}^2}. \tag{24}$$

Because of the method and renormalization scheme adopted in our calculations, our results are a bit different from that obtained in terms of the mass-insertion method. As a matter of fact, in our calculations, if the R-parity violating trilinear terms are also taken into account, their effects change the mass matrix of neutrino through their modifications to the effective interaction vertices and thus, as a reasonable approximation, can be included in the effective couplings of trilinear interaction vertices.

Now let us turn to the numerical computations. We diagonalize the mixing matrix and obtain the neutrino mass eigenvalues of three generations as well as the modified neutralino masses. This is a generalized see-saw mechanism.

Numerically, we adopt the basis with $\upsilon_{\tilde{\nu}_i} = 0.$ (i=1, 2, 3) and input the soft-breaking terms as

$$m_L^2(0,0) = -6.0 \times 10^6 \text{ GeV}^2, \quad m_L^2(1,1) = 6.0 \times 10^6 \text{ GeV}^2,$$

$$m_L^2(2,2) = 6.0 \times 10^6 \text{ GeV}^2, \quad m_L^2(3,3) = 6.0 \times 10^6 \text{ GeV}^2,$$

$$m_L^2(0,1) = 0. \text{ GeV}^2, \quad m_L^2(0,2) = 0. \text{ GeV}^2, \quad m_L^2(0,3) = 2.0 \times 10^5 \text{ GeV}^2,$$

$$m_L^2(1,2) = 0. \text{ GeV}^2, \quad m_L^2(1,3) = 0. \text{ GeV}^2, \quad m_L^2(2,3) = 0. \text{ GeV}^2.$$

We also set the R-parity conserving term

$$B^0 = 10^6 \text{ GeV}^2,$$

and the R-parity violation parameters B^i for three generations in the super-potential as

$$B^1 = B^2 = 10 \text{ GeV}^2 \quad ,$$

and B^3 is treated as a variable which can be fixed by comparing the calculated results with data. The choice of the soft breaking parameters in our scenario makes the lepton-number violation effects of the 1st and 2nd generation fermions are much less significant than that of the 3rd generation [17].

Several other concerned parameters are taken as

$$\tan\beta = 5, \;\; \mu^0 = 100 \text{ GeV}, \;\; \mu^i = 0 \;\; (i = 1, 2), \;\; \mu^3 = 10^{-6} \text{ GeV},$$

This choice leads to a nonzero mass less than 1 eV for the 3rd generation neutrino at the tree order.

Then we have obtained

$$m^{tree}_{\chi^0_1} \simeq 44.0 \text{ GeV},$$

$$m^{tree}_{\chi^0_2} \simeq 99.0 \text{ GeV},$$

$$m^{tree}_{\chi^0_3} \simeq 117.0 \text{ GeV},$$

$$m^{tree}_{\chi^0_4} \simeq 172.0 \text{ GeV}.$$

The corresponding dependence of $\Delta m_{23}^2 \equiv |m_{\nu_\tau}^2 - m_{\nu_\mu}^2|$ and $\Delta m_{12}^2 \equiv |m_{\nu_\mu}^2 - m_{\nu_e}^2|$ on the R-parity violation parameter of the third generation B^3 is shown in Fig.1. Simultaneously, we can have the mixing angles as

$$|\sin\theta_{23}| \simeq 0.26, \quad |\sin\theta_{12}| \simeq 0.034.$$

In the numerical analyses, we find that the mixing angles θ_{23}, θ_{12} do not vary much as the soft breaking parameter B_3 changes, and this small declination can be neglected in the physical discussions.

Therefore we can draw a rough conclusion that as the R-parity violation parameter of the third generation B^3 is larger than 10^5 GeV^2, the obtained neutrino mass difference Δm_{23}^2 and mixing angles can meet the data. From Fig.1, we obtain that when $B^3 > 10^5$ GeV, the neutrino mass differences

$$\Delta m_{23}^2 \sim 10^{-1}\ \mathrm{eV}^2 \quad \text{and} \quad \Delta m_{12}^2 \sim 10^{-6}\ \mathrm{eV}^2.$$

The recent data on the solar neutrino and atmospheric neutrino experiments have been given in [14]. The more precise values of the oscillation parameters at 90% c.l. are:

$$\Delta m_{\nu_\mu\nu_\tau}^2 \simeq (2\ -\ 8)\times 10^{-3}\ eV^2; \tag{25}$$

$$sin^2 2\theta_{\mu\tau} \simeq 0.8\ -\ 1$$

and the best fit to data for the solar neutrino experiments gives several possible solutions corresponding to various models as

$$VO : \Delta m^2 \simeq 6.5\times 10^{-11} eV^2; sin^2 2\theta \simeq 0.75 - 1 \tag{26}$$

$$SMA - MSW : \Delta m^2 \simeq 5\times 10^{-6} eV^2; sin^2 2\theta \simeq 5\times 10^{-3}$$

$$LMA - MSW : \Delta m^2 \simeq 1.2\times 10^{-5} - 3.1\times 10^{-4} eV^2; sin^2 2\theta \simeq 0.58\ -\ 1.00,$$

where "VO" means "vacuum oscillation" solution, "SMA-MSW" denotes the solution of Mikheev-Smirnov-Wolfenstein with small mixing angle whereas "LMA-MSW" is similar to "SMA-MSW" but with larger mixing angle.

Our results for Δm_{23}^2 is about 10^{-2} eV^2 for $B^3 > 3\times 10^5$ GeV^2 and Δm_{12}^2 is 10^{-5} eV^2 at the scale. The mixing angle is $\sin^2 2\theta_{23} \sim 0.27$ which is a bit smaller than the fitted data, but has the same order of magnitude. Due to the relatively larger errors in both experiments and theory, this difference is tolerable. The results of Δm_{12}^2, $\sin^2 2\theta_{21} \sim 4.6\times 10^{-3}$ in our calculation are consistent with the case of SMA-MSW very well.

4 Conclusion and discussion

In this work, we employ the on-shell renormalization scheme. We find that the supersymmetric model with L-number violation terms can result in the expected values which are gained by fitting the solar and atmospheric neutrino experiments. Even though the results are qualitatively consistent with that in the $\overline{MS}$ scheme, quantitatively they are a bit different from the later. To our understanding it is because the on-shell renormalization scheme is an over-subtraction scheme, after eliminating the divergence of the loop integrations, at any concerned order of perturbation (the one-loop order in our case), the equation of motion of the particle still holds, i.e. the mass-shell condition is respected. Instead, in the $\overline{MS}$ scheme, the mass-shell condition is violated at the concerned order of perturbation, and this may be the reason of the difference in the two schemes.

When we take into account the contribution from the R-parity violating trilinear terms in our numerical analysis, the available parameter space would be enlarged, it is easy to find a suitable subspace of concerned parameters which can fit all present experimental data. In our future studies, we may extend our analysis to other phenomenology where the R-parity violating terms play important roles and then, maybe, the available parameter space is more constrained and existence of both the bilinear and trilinear terms would be necessary. By comparison, in this work, their effects merge into the effective couplings of the vertices for the bilinear interactions, so that do not manifest themselves directly.

More studies on this subject need to be carried out, especially the parameter space for SUSY without R-parity is tremendously large, thus more precise measurements on neutrino oscillation as well as other rare B-decays may help to understand both neutrino physics and SUSY without R-parity.

Acknowledgements

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Appendix

A The definition of F_{2a}, F_{2b}, F_{3a}, F_{3b}

The functions that appear in the text are defined as

$$
\begin{aligned}
F_{2a}(m_1,m_2) &= (\mu_w)^{2\epsilon}\int_0^1 dx\int\frac{d^DQ}{(2\pi)^D}\frac{1}{(Q^2+xm_1^2+(1-x)m_2^2)^2} \\
&= \frac{1}{(4\pi)^2}\Big\{\frac{1}{\epsilon}-\gamma_E+\frac{m_1^2\ln\frac{4\pi\mu_w^2}{m_1^2}-m_2^2\ln\frac{4\pi\mu_w^2}{m_2^2}}{m_1^2-m_2^2}\Big\}, \\
F_{2b}(m_1,m_2) &= (\mu_w)^{2\epsilon}\int_0^1 dx\int\frac{d^DQ}{(2\pi)^D}\frac{1-x}{(Q^2+xm_1^2+(1-x)m_2^2)^2} \\
&= \frac{1}{2(4\pi)^2}\Big\{\frac{1}{\epsilon}-\gamma_E+\frac{m_1^2-3m_2^2}{2(m_1^2-m_2^2)} \\
&\quad +\frac{1}{(m_1^2-m_2^2)^2}\Big[m_1^2(m_1^2-2m_2^2)\ln\frac{4\pi\mu_w^2}{m_1^2}+m_2^2\ln\frac{4\pi\mu_w^2}{m_2^2}\Big]\Big\}, \\
F_{3a}(m_1,m_2) &= (\mu_w)^{2\epsilon}\int_0^1 dx\int\frac{d^DQ}{(2\pi)^D}\frac{2x(1-x)}{(Q^2+xm_1^2+(1-x)m_2^2)^3} \\
&= \frac{1}{(4\pi)^2}\frac{1}{(m_1^2-m_2^2)^3}\Big\{-m_1^2m_2^2\ln\frac{m_1^2}{m_2^2}+\frac{m_1^4-m_2^4}{2}\Big\}, \\
F_{3b}(m_1,m_2) &= (\mu_w)^{2\epsilon}\int_0^1 dx\int\frac{d^DQ}{(2\pi)^D}\frac{2x(1-x)^2}{(Q^2+xm_1^2+(1-x)m_2^2)^3} \\
&= \frac{1}{(4\pi)^2}\frac{1}{(m_1^2-m_2^2)^4}\Big\{\frac{1}{3}m_1^6+\frac{1}{6}m_2^6+\frac{1}{2}m_1^4m_2^2-m_1^2m_2^4-m_1^4m_2^2\ln\frac{m_1^2}{m_2^2}\Big\}.
\end{aligned}
\tag{27}
$$

B The 1-loop self energy diagrams of neutralino-neutrino mixing

- The internal particles are Z ~gauge boson and neutralinos (neutrinos through mixing) κ_α^0 ($\alpha = 1,2,\cdots,7$). The coupling constants can be written as

$$
\begin{aligned}
A_-^{(Z,\kappa_\alpha^0)} &= \frac{e}{4s_wc_w}\Big\{Z_N^{*\alpha,3}Z_N^{i,3}-\sum_{\delta=4}^{7}Z_N^{*\alpha,\delta}Z_N^{i,\delta}\Big\}, \\
A_+^{(Z,\kappa_\alpha^0)} &= -A_-^{(Z,\kappa_\alpha^0)}, \\
B_-^{(Z,\kappa_\alpha^0)} &= \frac{e}{4s_wc_w}\Big\{Z_N^{*j,3}Z_N^{\alpha,3}-\sum_{\delta=4}^{7}Z_N^{*j,\delta}Z_N^{\alpha,\delta}\Big\}, \\
B_+^{(Z,\kappa_\alpha^0)} &= -B_-^{(Z,\kappa_\alpha^0)}, \\
m_f &= m_{\kappa_\alpha^0}, \qquad\qquad m_V = m_z.
\end{aligned}
\tag{28}
$$

Z_N is the transformation matrix of neutrino-neutralino from the interaction basis to the mass basis. Corrections to the mass matrix are:

$$\delta m_{ij}^{(Z,\kappa^0)} = \sum_{\alpha=1}^{7}\Big\{\Big(12m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree}A_+^{(Z,\kappa_\alpha^0)}B_-^{(Z,\kappa_\alpha^0)} + ((m_{\kappa_i^0}^{tree})^2 + m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree}$$
$$+(m_{\kappa_0^j}^{tree})^2)A_-^{(Z,\kappa_\alpha^0)}B_+^{(Z,\kappa_\alpha^0)}\Big)m_{\kappa_\alpha^0}F_{3a}(m_{\kappa_\alpha^0}, m_Z)$$
$$-2\Big[((m_{\kappa_i^0}^{tree})^2m_{\kappa_j^0}^{tree} + 3m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2)A_-^{(Z,\kappa_\alpha^0)}B_-^{(Z,\kappa_\alpha^0)}$$
$$+(m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2 + 3(m_{\kappa_i^0}^{tree})^2m_{\kappa_j^0}^{tree})A_+^{(Z,\kappa_\alpha^0)}B_+^{(Z,\kappa_\alpha^0)}\Big]F_{3b}(m_{\kappa_\alpha^0}, m_Z)\Big] \tag{29}$$

- The internal particles are W ∼gauge boson and charginos (charged leptons) κ_α^0 ($\alpha = 1, 2, \cdots, 5$). The coupling constants can be written as

$$A_-^{(W,\kappa_\alpha^-)} = \frac{e}{s_w}\Big\{-Z_+^{*\alpha,1}Z_N^{i,2} + \frac{1}{\sqrt{2}}Z_+^{*\alpha,2}Z_N^{i,3}\Big\}$$
$$A_+^{(W,\kappa_\alpha^-)} = \frac{e}{s_w}\Big\{Z_-^{\alpha,1}Z_N^{*i,2} + \frac{1}{\sqrt{2}}\sum_{\delta=0}^{3}Z_-^{\alpha,2+\delta}Z_N^{*i,4+\delta}\Big\}$$
$$B_-^{(W,\kappa_\alpha^-)} = \frac{e}{s_w}\Big\{-Z_+^{\alpha,1}Z_N^{*j,2} + \frac{1}{\sqrt{2}}Z_+^{\alpha,2}Z_N^{*j,3}\Big\}$$
$$B_+^{(W,\kappa_\alpha^-)} = \frac{e}{s_w}\Big\{Z_-^{*\alpha,1}Z_N^{j,2} + \frac{1}{\sqrt{2}}\sum_{\delta=0}^{3}Z_-^{*\alpha,2+\delta}Z_N^{j,4+\delta}\Big\}$$
$$m_f = m_{\kappa_\alpha^-}, \qquad\qquad m_V = m_w \tag{30}$$

where $Z_\pm$ is the transformation matrix of charged lepton-chargino from the interaction basis to the mass basis. The W-chargino loop corrections to the mass matrix are:

$$\delta m_{ij}^{(W,\kappa^0)} = \sum_{\alpha=1}^{5}\Big\{\Big[12m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree}A_+^{(W,\kappa_\alpha^-)}B_-^{(W,\kappa_\alpha^-)} + ((m_{\kappa_i^0}^{tree})^2 + (m_{\kappa_j^0}^{tree})^2$$
$$+m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree})A_-^{(W,\kappa_\alpha^-)}B_+^{(W,\kappa_\alpha^-)}\Big]m_{\kappa_\alpha^-}F_{3a}(m_{\kappa_\alpha^-}, m_W)$$
$$+\Big[((m_{\kappa_i^0}^{tree})^2m_{\kappa_j^0}^{tree} + 3m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2)A_-^{(W,\kappa_\alpha^-)}B_-^{(W,\kappa_\alpha^-)}$$
$$+(3(m_{\kappa_i^0}^{tree})^2m_{\kappa_j^0}^{tree} + m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2)A_+^{(W,\kappa_\alpha^-)}B_+^{(W,\kappa_\alpha^-)}\Big]F_{3b}(m_{\kappa_\alpha^-}, m_W)\Big\} \tag{31}$$

- The internal particles are CP-even Higgs bosons H_β^0 ($\beta = 1, 2, \cdots, 5$) and neutralinos (neutrinos through mixing) κ_α^0 ($\alpha = 1, 2, \cdots, 7$). The coupling constants can be written as

$$A_-^{(H_\beta^0,\kappa_\alpha^0)} = \frac{e}{2s_wc_w}C_{snn}^{\alpha j\beta}$$

$$
\begin{aligned}
A_{+}^{(H_{\beta}^{0},\kappa_{\alpha}^{0})} &= \frac{e}{2s_w c_w} C_{snn}^{*\alpha j\beta} \\
B_{-}^{(H_{\beta}^{0},\kappa_{\alpha}^{0})} &= \frac{e}{2s_w c_w} C_{snn}^{\alpha\beta i} \\
B_{+}^{(H_{\beta}^{0},\kappa_{\alpha}^{0})} &= \frac{e}{2s_w c_w} C_{snn}^{*\alpha\beta i} \\
m_S = m_{H_{\beta}^{0}}, &\qquad\qquad m_f = m_{\kappa_{\alpha}^{0}}
\end{aligned} \tag{32}
$$

with

$$
C_{snn}^{\alpha\beta\gamma} = \Big(c_w Z_N^{\beta 2} - s_w Z_N^{\beta 1}\Big)\Big(\sum_{\delta=1}^{5} Z_E^{\alpha\delta} Z_N^{\gamma 2+\delta} - 2 Z_E^{\alpha 2} Z_N^{\gamma 3}\Big) \tag{33}
$$

and Z_E is the mixing matrix of CP-even Higgs bosons. The CP-even Higgs-neutralino loop corrections to the mass matrix are

$$
\begin{aligned}
\delta m_{ij}^{(H^0,\kappa^0)} = -\sum_{\alpha=1}^{7}\sum_{\beta=1}^{5} \bigg\{ & \Big(3 m_{\kappa_i^0}^{tree} m_{\kappa_j^0}^{tree} A_{-}^{(H_{\beta}^{0},\kappa_{\alpha}^{0})} B_{-}^{(H_{\beta}^{0},\kappa_{\alpha}^{0})} + ((m_{\kappa_i^0}^{tree})^2 + (m_{\kappa_j^0}^{tree})^2 \\
& + m_{\kappa_i^0}^{tree} m_{\kappa_j^0}^{tree}) A_{+}^{(H_{\beta}^{0},\kappa_{\alpha}^{0})} B_{+}^{(H_{\beta}^{0},\kappa_{\alpha}^{0})}\Big) m_{\kappa_{\alpha}^{-}} F_{3a}(m_{\kappa_{\alpha}^{0}}, m_{H_{\beta}^{0}}) \\
& + \Big(((m_{\kappa_i^0}^{tree})^2 m_{\kappa_j^0}^{tree} + 3 m_{\kappa_i^0}^{tree} (m_{\kappa_j^0}^{tree})^2) A_{+}^{(H_{\beta}^{0},\kappa_{\alpha}^{0})} B_{-}^{(H_{\beta}^{0},\kappa_{\alpha}^{0})} \\
& + (3 (m_{\kappa_i^0}^{tree})^2 m_{\kappa_j^0}^{tree} + m_{\kappa_i^0}^{tree} (m_{\kappa_j^0}^{tree})^2) A_{-}^{(H_{\beta}^{0},\kappa_{\alpha}^{0})} B_{+}^{(H_{\beta}^{0},\kappa_{\alpha}^{0})}\Big) F_{3b}(m_{\kappa_{\alpha}^{0}}, m_{H_{\beta}^{0}}) \bigg\}
\end{aligned} \tag{34}
$$

- The internal particles are CP-odd Higgs bosons A_{β}^{0} ($\beta = 1, 2, \cdots, 5$) and neutralinos (neutrinos through mixing) κ_{α}^{0} ($\alpha = 1, 2, \cdots, 7$). The coupling constants are

$$
\begin{aligned}
A_{-}^{(A_{\beta}^{0},\kappa_{\alpha}^{0})} &= i\frac{e}{2s_w c_w} C_{onn}^{\alpha j\beta} \\
A_{+}^{(A_{\beta}^{0},\kappa_{\alpha}^{0})} &= -i\frac{e}{2s_w c_w} C_{onn}^{*\alpha j\beta} \\
B_{-}^{(A_{\beta}^{0},\kappa_{\alpha}^{0})} &= i\frac{e}{2s_w c_w} C_{onn}^{\alpha\beta i} \\
B_{+}^{(A_{\beta}^{0},\kappa_{\alpha}^{0})} &= -i\frac{e}{2s_w c_w} C_{onn}^{*\alpha\beta i} \\
m_S = m_{A_{\beta}^{0}}, &\qquad\qquad m_f = m_{\kappa_{\alpha}^{0}}
\end{aligned} \tag{35}
$$

with

$$
C_{onn}^{\alpha\beta\gamma} = \Big(c_w Z_N^{\beta 2} - s_w Z_N^{\beta 1}\Big)\Big(\sum_{\delta=1}^{5} Z_O^{\alpha\delta} Z_N^{\gamma 2+\delta} - 2 Z_O^{\alpha 2} Z_N^{\gamma 3}\Big) \tag{36}
$$

and Z_O is the mixing matrix of CP-odd Higgs bosons and neutral particles. The CP-odd Higgs-neutralino loop corrections to the mass matrix are

$$\delta m_{ij}^{(A^0,\kappa^0)} = -\sum_{\alpha=1}^{7}\sum_{\beta=1}^{5}\Big\{\Big[3m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree}A_-^{(A_\beta^0,\kappa_\alpha^0)}B_-^{(A_\beta^0,\kappa_\alpha^0)} + ((m_{\kappa_i^0}^{tree})^2 + (m_{\kappa_j^0}^{tree})^2$$
$$+m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree})A_+^{(A_\beta^0,\kappa_\alpha^0)}B_+^{(A_\beta^0,\kappa_\alpha^0)}\Big]m_{\kappa_\alpha^-}F_{3a}(m_{\kappa_\alpha^0}, m_{A_\beta^0})$$
$$+\Big[((m_{\kappa_i^0}^{tree})^2 m_{\kappa_j^0}^{tree} + 3m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2)A_+^{(A_\beta^0,\kappa_\alpha^0)}B_-^{(A_\beta^0,\kappa_\alpha^0)}$$
$$+(3(m_{\kappa_i^0}^{tree})^2 m_{\kappa_j^0}^{tree} + m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2)A_-^{(A_\beta^0,\kappa_\alpha^0)}B_+^{(A_\beta^0,\kappa_\alpha^0)}\Big]F_{3b}(m_{\kappa_\alpha^0}, m_{A_\beta^0})\Big\}. \tag{37}$$

- The internal particles are charged Higgs bosons H_β^+ ($\beta = 1, 2, \cdots, 8$) and charginos (charged leptons) κ_α^- ($\alpha = 1, 2, \cdots, 5$). The coupling constants can be written as

$$A_-^{(H_\beta^+,\kappa_\alpha^-)} = -\frac{e}{s_w c_w}C_{Rnk}^{*\beta\alpha j}$$
$$A_+^{(H_\beta^+,\kappa_\alpha^-)} = \frac{e}{s_w c_w}C_{Lnk}^{*\beta\alpha j}$$
$$B_-^{(H_\beta^+,\kappa_\alpha^-)} = \frac{e}{s_w c_w}C_{Lnk}^{\beta\alpha i}$$
$$B_+^{(H_\beta^+,\kappa_\alpha^-)} = -\frac{e}{s_w c_w}C_{Rnk}^{\beta\alpha i}$$
$$m_S = m_{H_\beta^-}, \qquad m_f = m_{\kappa_\alpha^-}. \tag{38}$$

The symbols C_{Lnk}, C_{Rnk} are defined as

$$C_{Lnk}^{\alpha\beta\gamma} = \Big\{\sum_{\delta=2} Z_C^{\alpha\delta}\Big[\frac{1}{\sqrt{2}}\Big(c_w Z_-^{\beta\delta}Z_N^{\gamma 2} + s_w Z_-^{\beta\delta}Z_N^{\gamma 1}\Big) - c_w Z_-^{\beta 1}Z_N^{\gamma(2+\delta)}\Big]$$
$$+\sum_{IJK}\frac{\lambda_{IJK}}{2es_w c_w}\Big[Z_C^{\alpha(5+I)}Z_-^{\beta(2+J)}Z_N^{\gamma(4+K)} - Z_C^{\alpha(5+I)}Z_-^{\beta(2+K)}Z_N^{\gamma(4+J)}\Big]\Big\}$$
$$C_{Rnk}^{\alpha\beta\gamma} = \Big\{Z_C^{\alpha 2}\Big[\frac{1}{\sqrt{2}}\Big(c_w Z_+^{*\beta 2}Z_N^{*\gamma 2} + s_w Z_+^{*\beta 2}Z_N^{*\gamma 1}\Big) + c_w Z_+^{*\beta 1}Z_N^{\gamma 3}\Big] + \sqrt{2}s_w\sum_{\delta=3}^{5}Z_C^{\alpha(5+\delta)}Z_+^{*\beta(2+\delta)}Z_N^{*\gamma 1}\Big\}$$
$$+\sum_{IJK}\frac{\lambda_{IJK}}{2es_w c_w}\Big[Z_C^{\alpha(2+I)}Z_-^{*\beta(4+J)}Z_N^{*\gamma(2+K)} - Z_C^{\alpha(2+J)}Z_-^{*\beta(4+I)}Z_N^{*\gamma(2+K)}\Big] \tag{39}$$

and Z_C is the mixing matrix of charged Higgs bosons and sleptons. The Charged Higgs-chargino loop corrections to the mass matrix are

$$\delta m_{ij}^{(H^+,\kappa^-)} = -\sum_{\alpha=1}^{7}\sum_{\beta=1}^{8}\Big\{\Big[3m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree}A_-^{(H_\beta^+,\kappa_\alpha^-)}B_-^{(H_\beta^+,\kappa_\alpha^-)} + (m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree} + (m_{\kappa_i^0}^{tree})^2$$

$$+(m_{\kappa_j^0}^{tree})^2)A_+^{(H_\beta^+,\kappa_\alpha^-)}B_+^{(H_\beta^+,\kappa_\alpha^-)}\Big]m_{\kappa_\alpha^-}F_{3a}(m_{\kappa_\alpha^-},m_{H_\beta^+})$$

$$+\Big[((m_{\kappa_i^0}^{tree})^2m_{\kappa_j^0}^{tree}+3m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2)A_+^{(H_\beta^+,\kappa_\alpha^-)}B_-^{(H_\beta^+,\kappa_\alpha^-)}+(3(m_{\kappa_i^0}^{tree})^2m_{\kappa_j^0}^{tree}$$

$$+m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2)A_-^{(H_\beta^+,\kappa_\alpha^-)}B_+^{(H_\beta^+,\kappa_\alpha^-)}\Big]F_{3b}(m_{\kappa_\alpha^-},m_{H_\beta^+})\Big\}. \tag{40}$$

- The internal particles are up-type scalar quarks $\tilde{U}_\alpha^k$ ($\alpha=1,2$; k=1, 2, 3.) and quark u^k. The coupling constants can be written as

$$A_-^{(\tilde{U}_\alpha^k,u^k)}=\frac{e}{\sqrt{2}s_wc_w}Z_{\tilde{U}^k}^{*\alpha1}\Big(c_wZ_N^{j2}+\frac{1}{3}s_wZ_N^{j1}\Big)-\frac{em_{u^k}}{2\sqrt{2}s_w\sin\beta m_w}Z_{\tilde{U}^k}^{*\alpha1}Z_N^{j3}$$

$$A_+^{(\tilde{U}_\alpha^k,u^k)}=\frac{2\sqrt{2}e}{3c_w}Z_{\tilde{U}^k}^{*\alpha2}Z_N^{*j1}-\frac{em_{u^k}}{2\sqrt{2}s_w\sin\beta m_w}Z_{\tilde{U}^k}^{*\alpha1}Z_N^{*j3}$$

$$B_-^{(\tilde{U}_\alpha^k,u^k)}=\frac{2\sqrt{2}e}{3c_w}Z_{\tilde{U}^k}^{\alpha2}Z_N^{i1}-\frac{em_{u^k}}{2\sqrt{2}s_w\sin\beta m_w}Z_{\tilde{U}^k}^{\alpha1}Z_N^{i3}$$

$$B_+^{(\tilde{U}_\alpha^k,u^k)}=\frac{e}{\sqrt{2}s_wc_w}Z_{\tilde{U}^k}^{\alpha1}\Big(c_wZ_N^{*i2}+\frac{1}{3}s_wZ_N^{*i1}\Big)-\frac{em_{u^k}}{2\sqrt{2}s_w\sin\beta m_w}Z_{\tilde{U}^k}^{\alpha1}Z_N^{*i3}$$

$$m_S=m_{\tilde{U}_\alpha^k},\qquad\qquad m_f=m_{u^k}, \tag{41}$$

where $Z_{\tilde{U}^k}$ denotes the mixing matrix of the k-th left-handed and right-handed up-type scalar quarks. The up-type scalar-quark-quark loop corrections to the mass matrix are

$$\delta m_{ij}^{(\tilde{U},u)}=-\sum_{\alpha=1}^{2}\sum_{k=1}^{3}\Big\{\Big[3m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree}A_-^{(\tilde{U}_\alpha^k,u^k)}B_-^{(\tilde{U}_\alpha^k,u^k)}+((m_{\kappa_i^0}^{tree})^2+(m_{\kappa_j^0}^{tree})^2$$

$$+m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree})A_+^{(\tilde{U}_\alpha^k,u^k)}B_+^{(\tilde{U}_\alpha^k,u^k)}\Big]m_{u^k}F_{3a}(m_{u^k},m_{\tilde{U}_\alpha^k})$$

$$+\Big[((m_{\kappa_i^0}^{tree})^2m_{\kappa_j^0}^{tree}+3m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2)A_+^{(\tilde{U}_\alpha^k,u^k)}B_-^{(\tilde{U}_\alpha^k,u^k)}+(m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2$$

$$+3(m_{\kappa_i^0}^{tree})^2m_{\kappa_j^0}^{tree})A_-^{(\tilde{U}_\alpha^k,u^k)}B_+^{(\tilde{U}_\alpha^k,u^k)}\Big]F_{3b}(m_{u^k},m_{\tilde{U}_\alpha^k})\Big\} \tag{42}$$

- The internal particles are down-type scalar quarks $\tilde{D}_\alpha^k$ ($\alpha=1,2$; k=1, 2, 3.) and quark d^k. The coupling constants are written as

$$A_-^{(\tilde{D}_\alpha^k,d^k)}=\frac{e}{\sqrt{2}s_wc_w}Z_{\tilde{D}^k}^{*\alpha1}\Big(-c_wZ_N^{j2}+\frac{1}{3}s_wZ_N^{j1}\Big)+\frac{1}{2}\sum_{K=0}^{3}\lambda_d^{Kkk}Z_{\tilde{D}^k}^{*\alpha2}Z_N^{j(4+K)},$$

$$A_+^{(\tilde{D}_\alpha^k,d^k)}=-\frac{\sqrt{2}e}{3c_w}Z_{\tilde{D}^k}^{*\alpha2}Z_N^{*j1}+\frac{1}{2}\sum_{K=0}^{3}\lambda_d^{Kkk}Z_{\tilde{D}^k}^{*\alpha1}Z_N^{*j(4+K)},$$

$$B_{+}^{(\tilde{D}_\alpha^k,d^k)}=-\frac{\sqrt{2}e}{3c_w}Z_{\tilde{D}^k}^{\alpha 2}Z_N^{i1}+\frac{1}{2}\sum_{K=0}^{3}\lambda_d^{Kkk}Z_{\tilde{D}^k}^{\alpha 1}Z_N^{i(4+K)},$$

$$B_{-}^{(\tilde{D}_\alpha^k,d^k)}=\frac{e}{\sqrt{2}s_wc_w}Z_{\tilde{D}^k}^{\alpha 1}\Big(-c_wZ_N^{*i2}+\frac{1}{3}s_wZ_N^{*i1}\Big)+\frac{1}{2}\sum_{K=0}^{3}\lambda_d^{Kkk}Z_{\tilde{D}^k}^{\alpha 2}Z_N^{i(4+K)}$$

$$m_S=m_{\tilde{D}_\alpha^k},\qquad\qquad m_f=m_{d^k},\tag{43}$$

where $Z_{\tilde{D}^k}$ denotes the mixing matrix of k-th left-handed and right-handed down-type scalar quarks. The up type scalar quark-quark loop corrections to the mass matrix are

$$\begin{aligned}\delta m_{ij}^{(\tilde{D},d)}=&-\sum_{\alpha=1}^{2}\sum_{k=1}^{3}\bigg\{\Big[3m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree}A_{-}^{(\tilde{D}_\alpha^k,d^k)}B_{-}^{(\tilde{D}_\alpha^k,d^k)}+((m_{\kappa_i^0}^{tree})^2+(m_{\kappa_j^0}^{tree})^2\\&+m_{\kappa_i^0}^{tree}m_{\kappa_j^0}^{tree})A_{+}^{(\tilde{D}_\alpha^k,d^k)}B_{+}^{(\tilde{D}_\alpha^k,d^k)}\Big]m_{d^k}F_{3a}(m_{d^k},m_{\tilde{D}_\alpha^k})\\&+\Big[(3(m_{\kappa_i^0}^{tree})^2m_{\kappa_j^0}^{tree}+m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2)A_{+}^{(\tilde{D}_\alpha^k,d^k)}B_{-}^{(\tilde{D}_\alpha^k,d^k)}+((m_{\kappa_i^0}^{tree})^2m_{\kappa_j^0}^{tree}\\&+3m_{\kappa_i^0}^{tree}(m_{\kappa_j^0}^{tree})^2)A_{-}^{(\tilde{D}_\alpha^k,d^k)}B_{+}^{(\tilde{D}_\alpha^k,d^k)}\Big]F_{3b}(m_{d^k},m_{\tilde{D}_\alpha^k})\bigg\}.\end{aligned}\tag{44}$$

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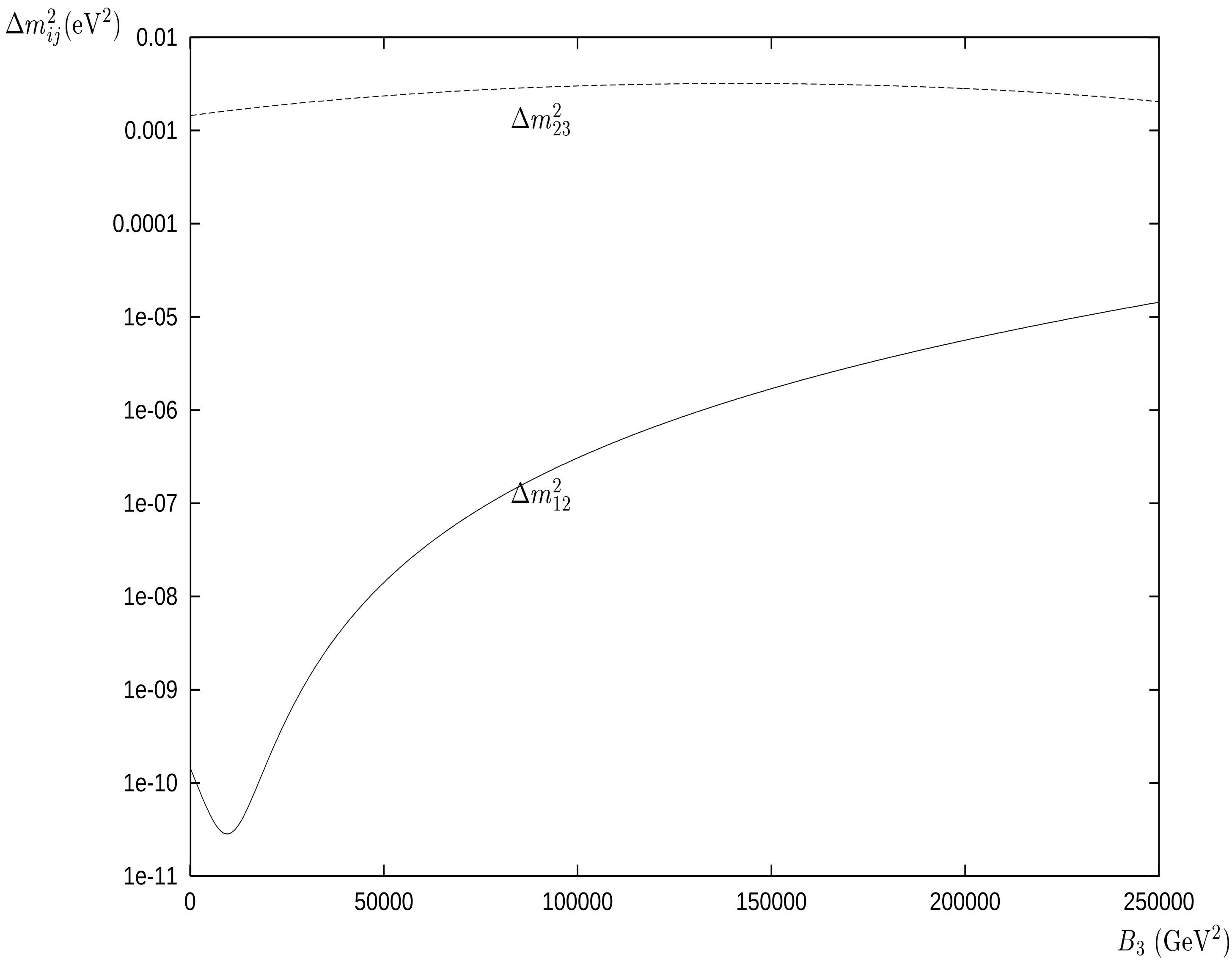

Figure 1: Dependence of Δm^2_{ij} on the soft R-parity violation parameter B^3 with (a)solid-line: Δm^2_{12} and (b)dash-line: Δm^2_{23}. The X-axis corresponds to the B^3 (in GeV^2) and the Y-axis corresponds to the Δm^2_{ij} (in eV^2). The other parameters are taken as in the text.